Multilevel Hierarchical Bayesian vs. State-Space Approach in a Time Series SAE Application: the Dutch Travel Survey (OViN)

Oksana Bollineni-Balabay, Jan van den Brakel, Franz Palm

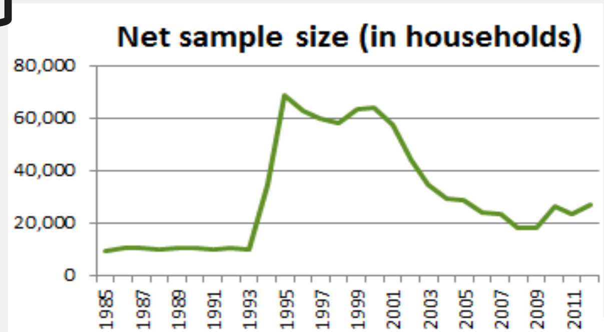
Statistics Netherlands

The Dutch Travel Survey (OViN)

- **annual** estimates for the **average number km per person per day (km/pppd)** for intersections of**:**
    - **7 motives:**
        - **Work, Business, Visits, Shopping,**
        - **Education, Recreative, Other;**
    - **8 transport modalities:**
        - **Car-driver, Car-passenger, Train,**
        - **Bus/Tram/Metro, Scooter, Bicycle,**
        - **Walking;**
    - **(12 provinces)**;

National level: M=**56** domains;
Provincial level: M=**672** domains

- 1985-2013;
- net sample size:
  10 000-68 000 households.



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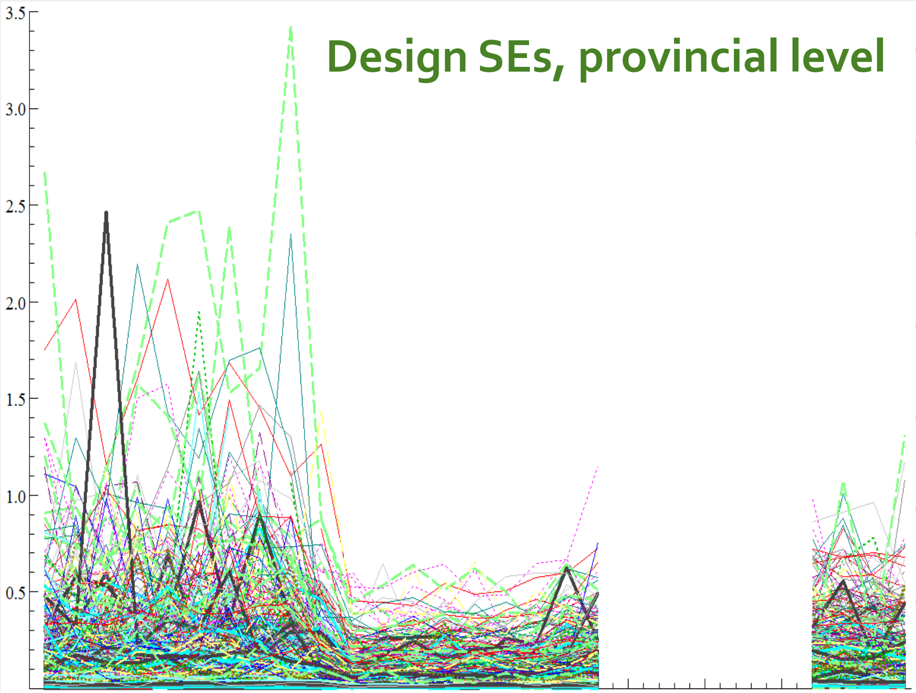
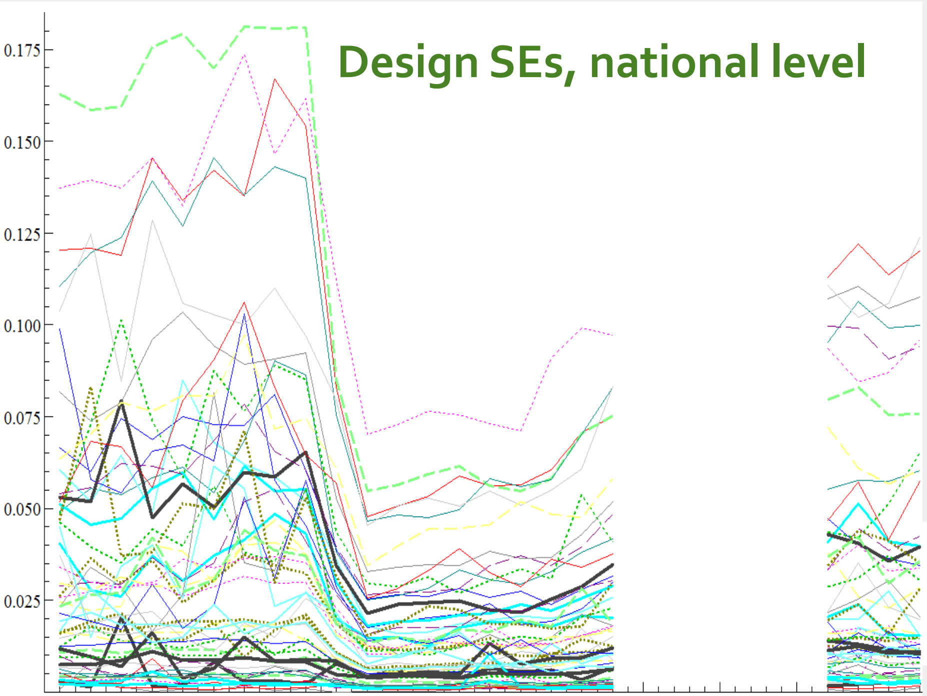
Survey redesigns

- before 1994, only residents >=12 years old;
- 2004-2009: data collection by another agency (MON);
- since 2010: conducted by Statistics Netherlands
- 1995: sample size $\times$ 6;





Problems and Tools

– Problems with the DTS (OViN):
  - small sample sizes;
  - discontinuities due to multiple survey redesigns.
– Solution: use the sample information over time and domain space:
  - **multilevel time series** models;
  - **structural time series** models with **unobserved components**.

Multilevel Model

design variance estimates,
assumed to be known

$Y\downarrow t = \theta\downarrow t + e\downarrow t, \quad e\downarrow t \sim IN\,(\mathbf{0},\Phi\downarrow t$

$\theta\downarrow t \quad = c\iota \; + \; \nu + X'\downarrow 1,t\uparrow \beta\downarrow 1 \; + \ldots + X\uparrow'\downarrow K,t\uparrow \beta\downarrow K + u\downarrow t,\; t \in \{1,\ldots,T\}$

time indicator

[■θ↓1,t @θ↓2,t @■⋮ M,t]=c[■1@1@■⋮@⋮@1]+[■ ν↓1 @ν↓2 @■⋮@⋮@ν↓M]+X′↓1,t↑ [■β↓1,1 @β↓1,2 @■⋮@⋮@β↓1,M]+X′↓K,t↑ [■0@0@■⋮@β↓K,m @■β↓K,m +1 @■⋮@■0@⋮]+[■u↓1,t @u↓2,t @■.@■.@⋮ @u↓M,t]

area REs
$\sim IIN(0,$
$\sigma\downarrow v\uparrow 2\,)$

β↓1994

β↓2010

β↓2004 *Recreative*
β↓2004 *Other motives*

stochastic trend

Structural Time Series Model

$$Y_{m,t} = \theta_{m,t} + e_{m,t}, \quad e_{m,t} \sim IN\ (0, \mathrm{Var}(Y_{m,t}))$$

$$\theta_{m,t} = \boldsymbol{X}'_{\boldsymbol{m,t}} \boldsymbol{\beta}_{\boldsymbol{m}} + L_{m,t} + \varepsilon_{m,t}, \quad \varepsilon_{m,t} \sim IIN\ (0, \sigma_{\varepsilon}^{2}), t \in \{1, \ldots, T\}$$

$$L_{m,t} = u_{m,t} + c + \nu_m$$

$$Y_{m,t} = \boldsymbol{X}'_{\boldsymbol{m,t}} \boldsymbol{\beta}_{\boldsymbol{m}} + \overbrace{L_{m,t} + \varepsilon_{m,t}} + e_{m,t},$$

composite error term:

$$e_{m,t} = k_{m,t} \sqrt{\mathrm{Var}(Y_{m,t})},$$

$$k_{m,t} \sim IIN(0, \sigma_k^{2}),$$

$$\sigma_k^{2} \approx 1$$

Smooth Trend Model

$$\boldsymbol{u_t = u_{t-1} + r_{t-1}}$$

$$\boldsymbol{r_t = r_{t-1} + \epsilon_t},$$

$$\boldsymbol{\epsilon_t \sim IIN(0, \Sigma_u)}, t \in \{1,\dots,T\}$$

$$\begin{aligned}
\boldsymbol{\Sigma_u = diag}(&\sigma_{p_1 Mot_1 Mod_1}^2\ \sigma_{p_1 Mot_1 Mod_2}^2 \dots \sigma_{p_1 Mot_1 Mod_8}^2 \\
&\sigma_{p_1 Mot_2 Mod_1}^2\ \sigma_{p_1 Mot_2 Mod_2}^2 \dots \sigma_{p_1 Mot_2 Mod_8}^2 \\
&\dots \\
&\sigma_{p_{12} Mot_7 Mod_1}^2\ \sigma_{p_1 Mot_7 Mod_2}^2 \dots \sigma_{p_1 Mot_7 Mod_8}^2)
\end{aligned}$$

Model Estimation

Multilevel Models:

– Hierarchical Bayesian (**HB**) approach with Gibbs-sampler, *mcmcsae* R-package (Boonstra, 2015);
– non-informative priors;
– Gelman-Rubin convergence statistic ($\hat{R}$) for each parameter;
– DIC criterion for model selection;
– informal criteria (e.g., adequacy of point-estimates).

Structural Time Series (STS) Model:

– ML approach for hyperparameters;
– put into a state-space form; the Kalman filter for state estimates;
– OxMetrics 7, SsfPack 3.0

Hyperparameter Estimation

STS model:

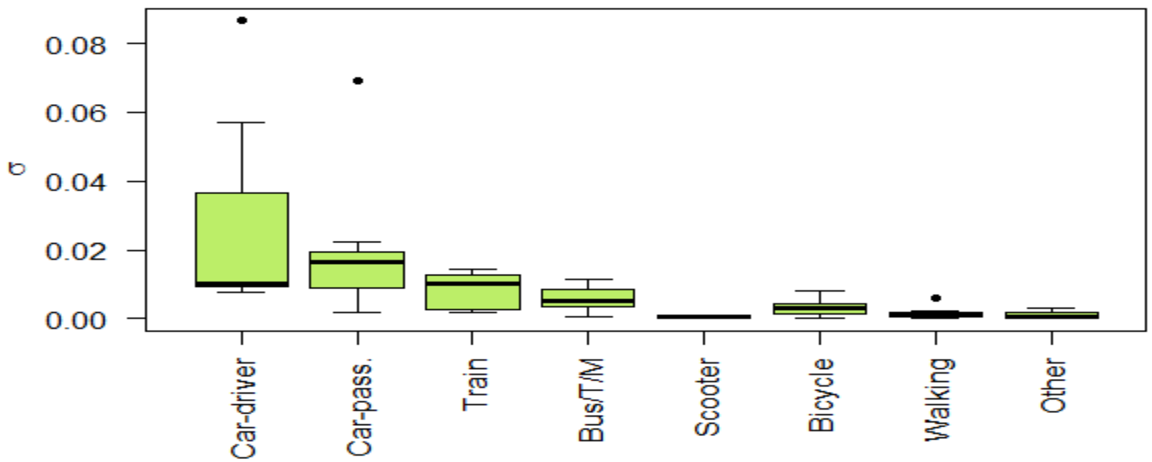
– $(\sigma_{p_1 Mot_1 Mod_1}^2 \ldots \sigma_{p_{12} Mot_7 Mod_8}^2, \sigma_k^2)$ –ML-estimator;

Multilevel HB model:

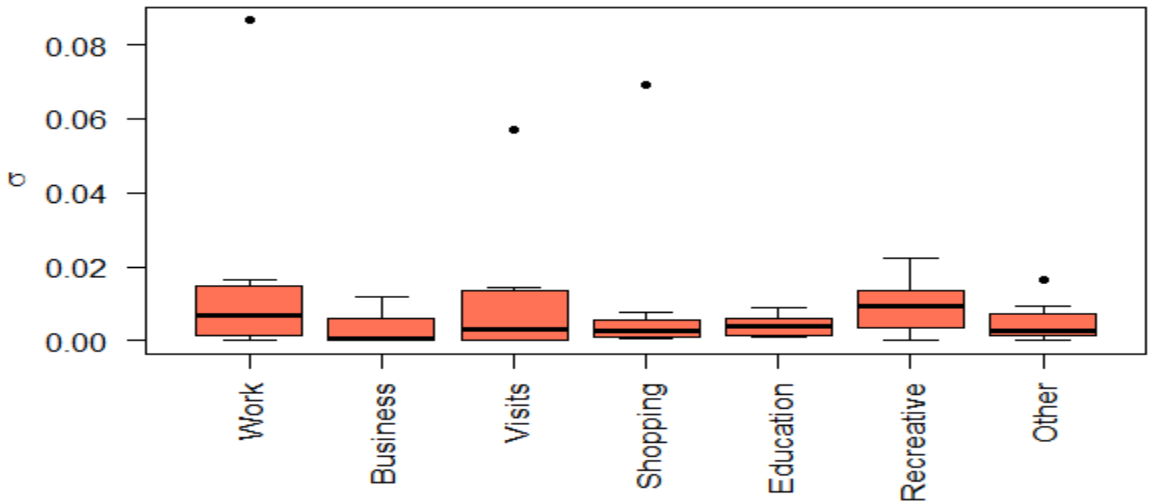
– $(\sigma_{p_1 Mot_1 Mod_1}^2 \ldots \sigma_{p_{12} Mot_7 Mod_8}^2, \sigma_v^2)$ – diffuse *Inv*-χ^2 priors

Pooling σ_u^2 across Motives/ Mod

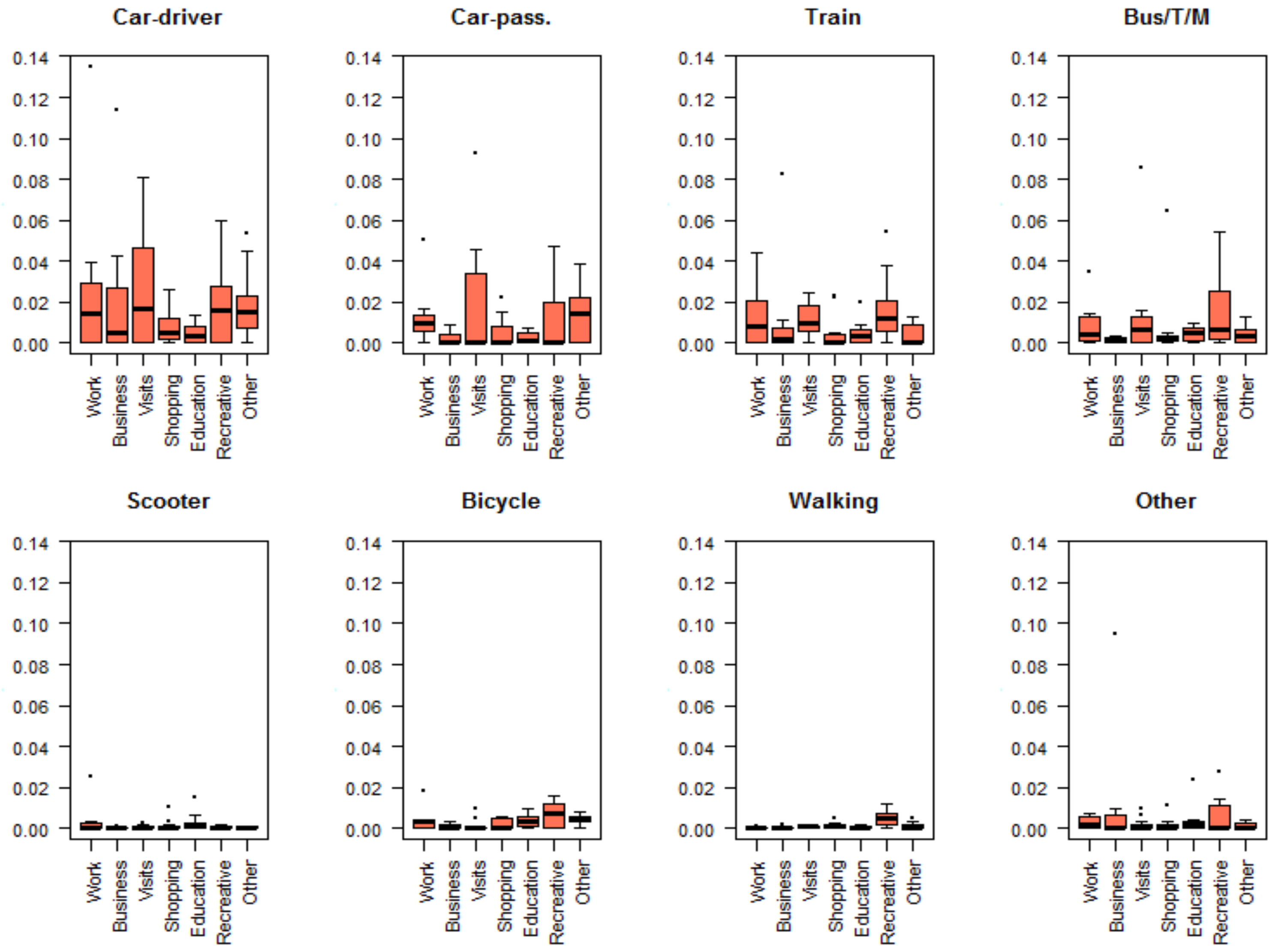
National level, Modalities, σ_u



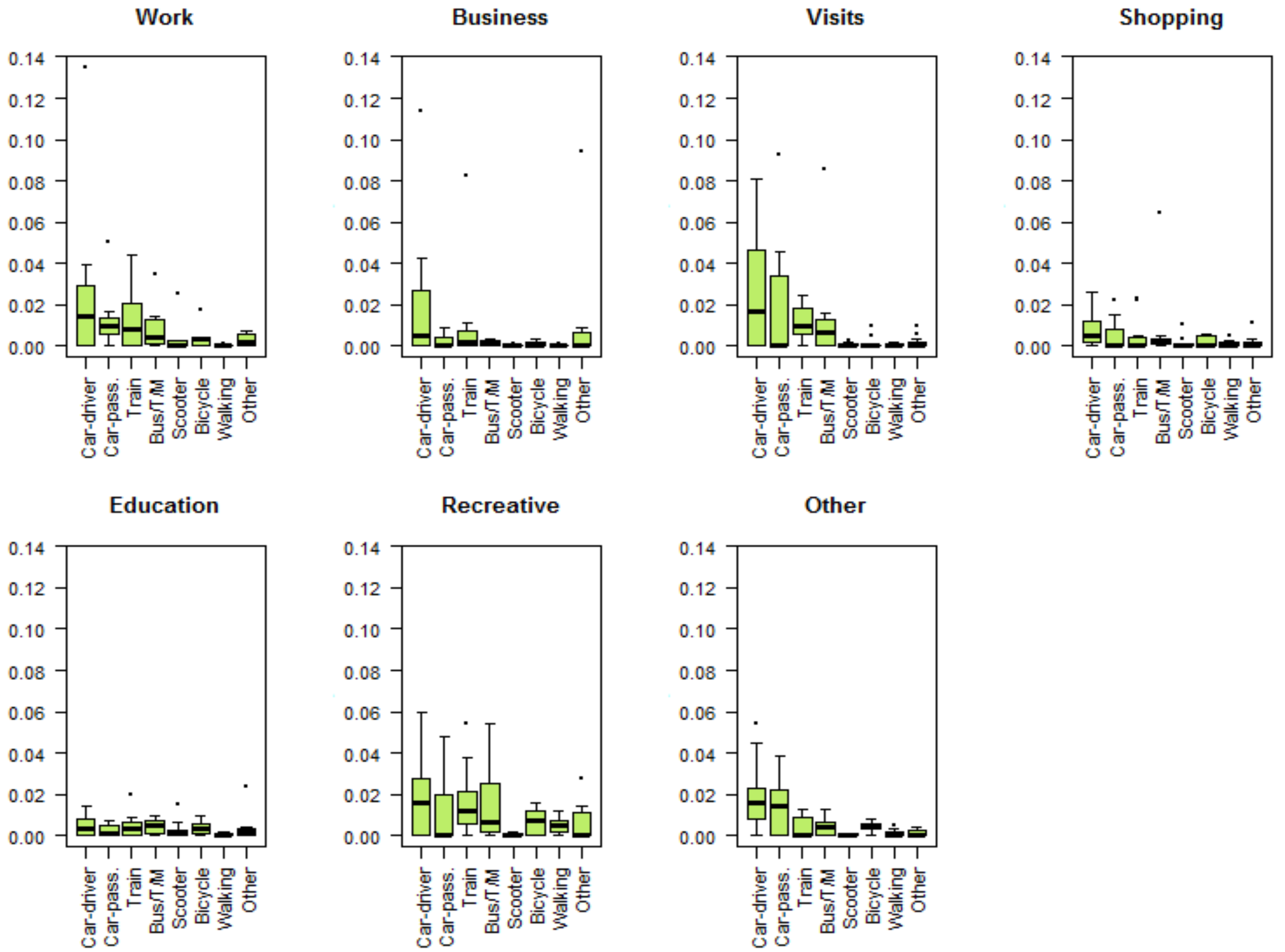
National level, Motives, σ_u



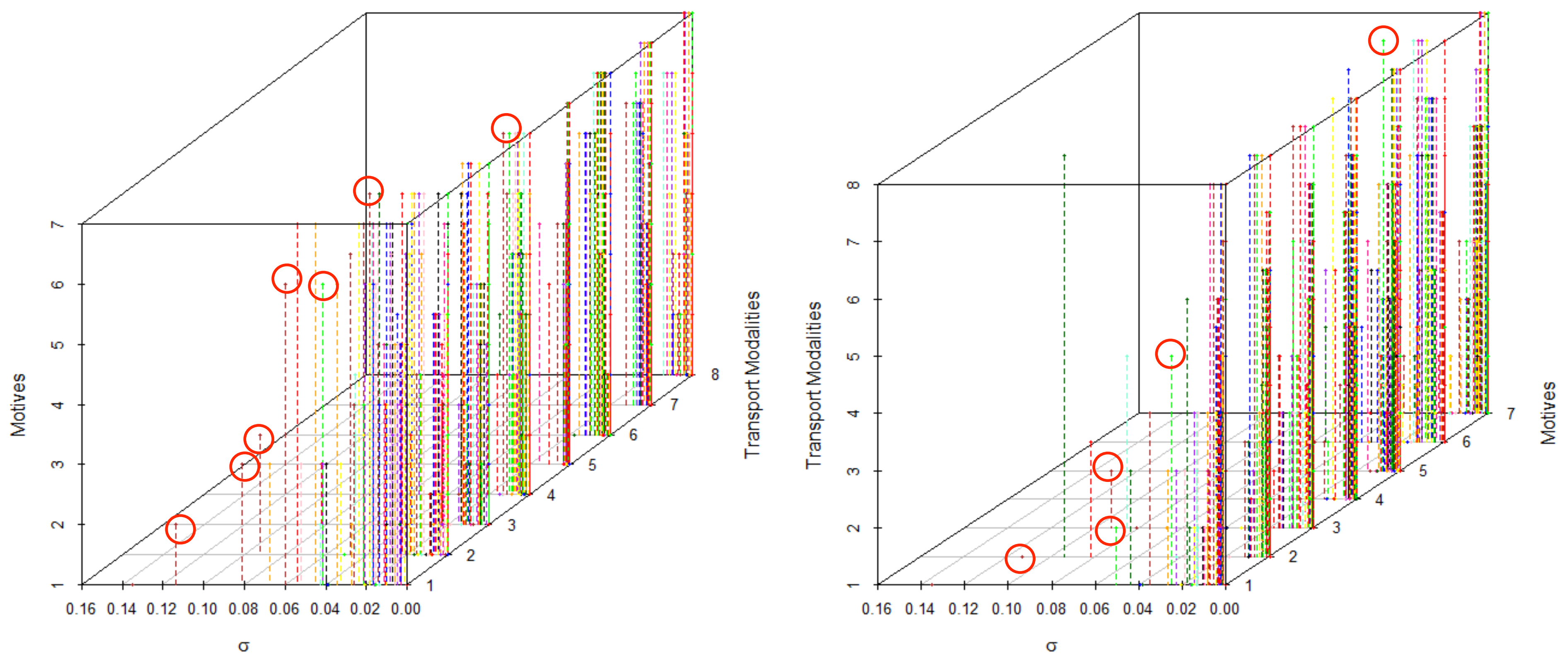
Pooling across Motives?

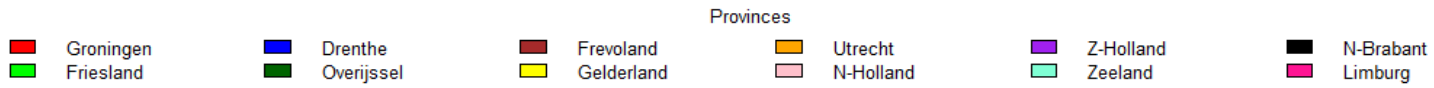


Pooling σ_u^2 across Modalities?



Pooling σ_{u}^{2} across Provinces?





Pooling $\sigma\downarrow u\uparrow 2$

- $\sigma\downarrow u\uparrow 2$ are too different to be pooled across **motives** and **modalities**;

- $\sigma\downarrow u\uparrow 2$ can be pooled across **provinces** within two groups:
  - Frevoland and Friesland;
  - the other 10 provinces

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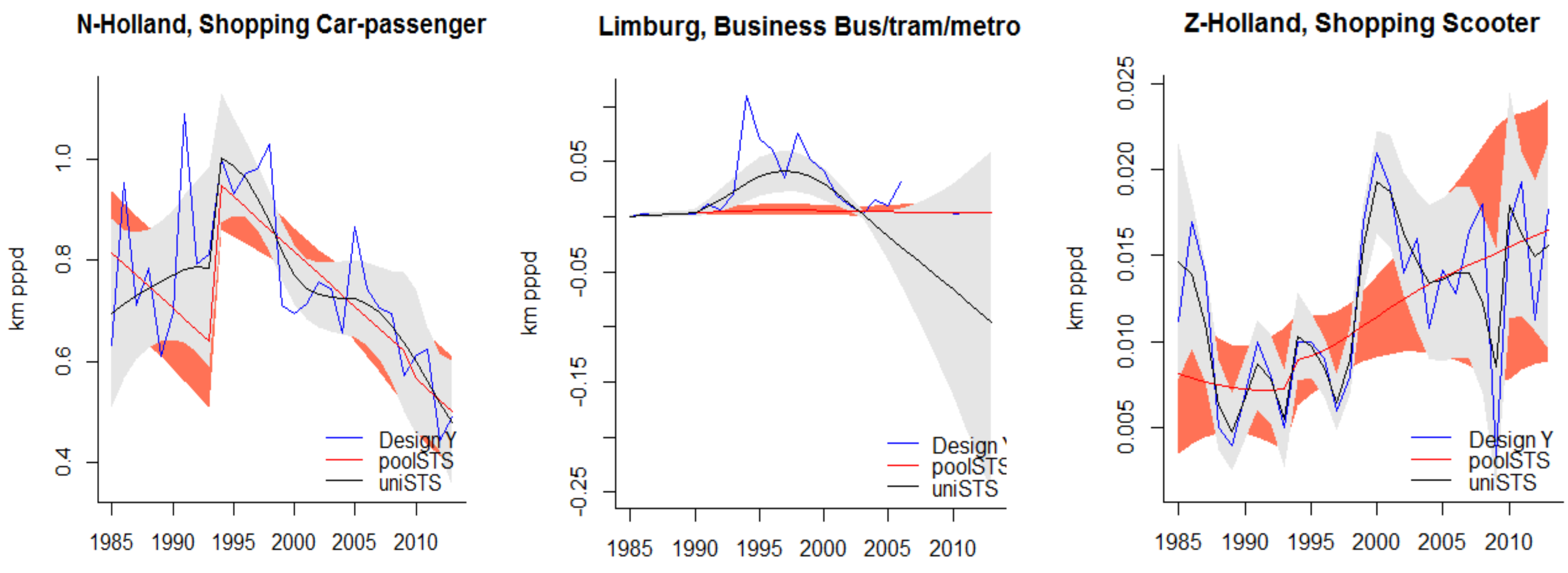
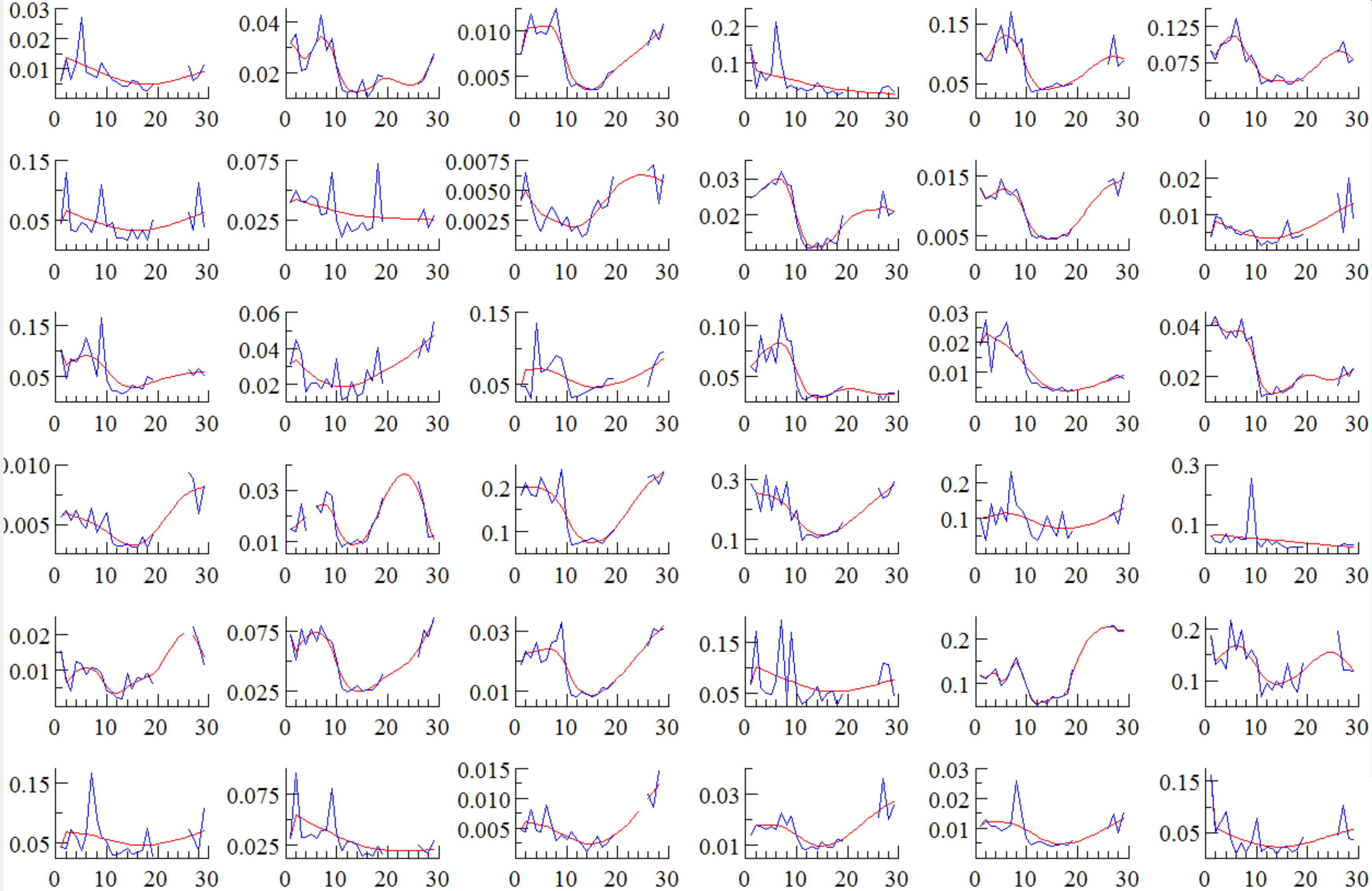
Pro’s and Con’s of Pooling

– Con’s of pooling:
  - provinces too different to be pooled together (Fig.1);
– Pro’s of pooling:
  - idiosyncrasies in PEs and design SEs ruled out (Fig.2,3);

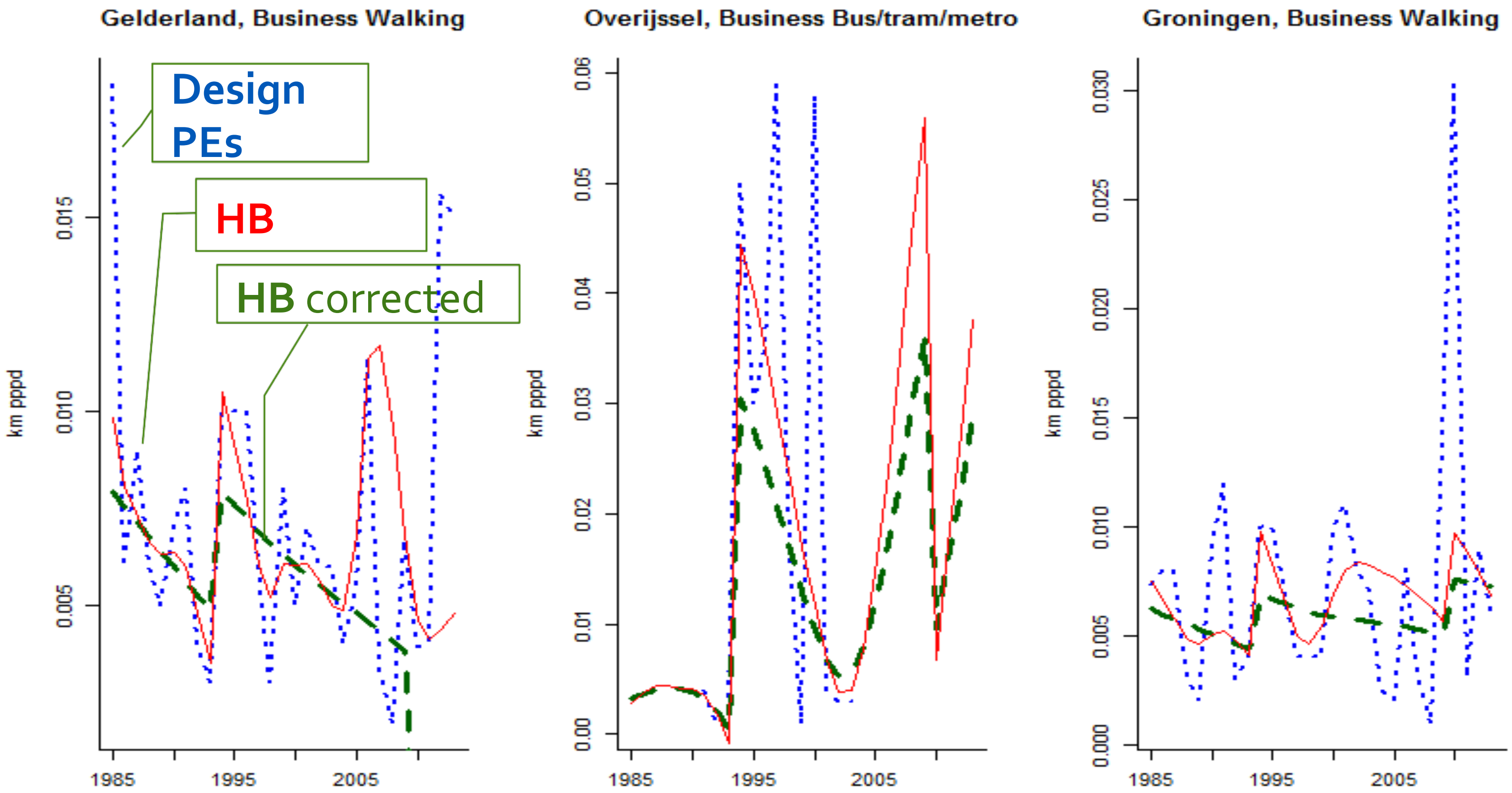


Tackling Unreliable Design SE Estimates



Tackling Unreliable Design SE Estimates

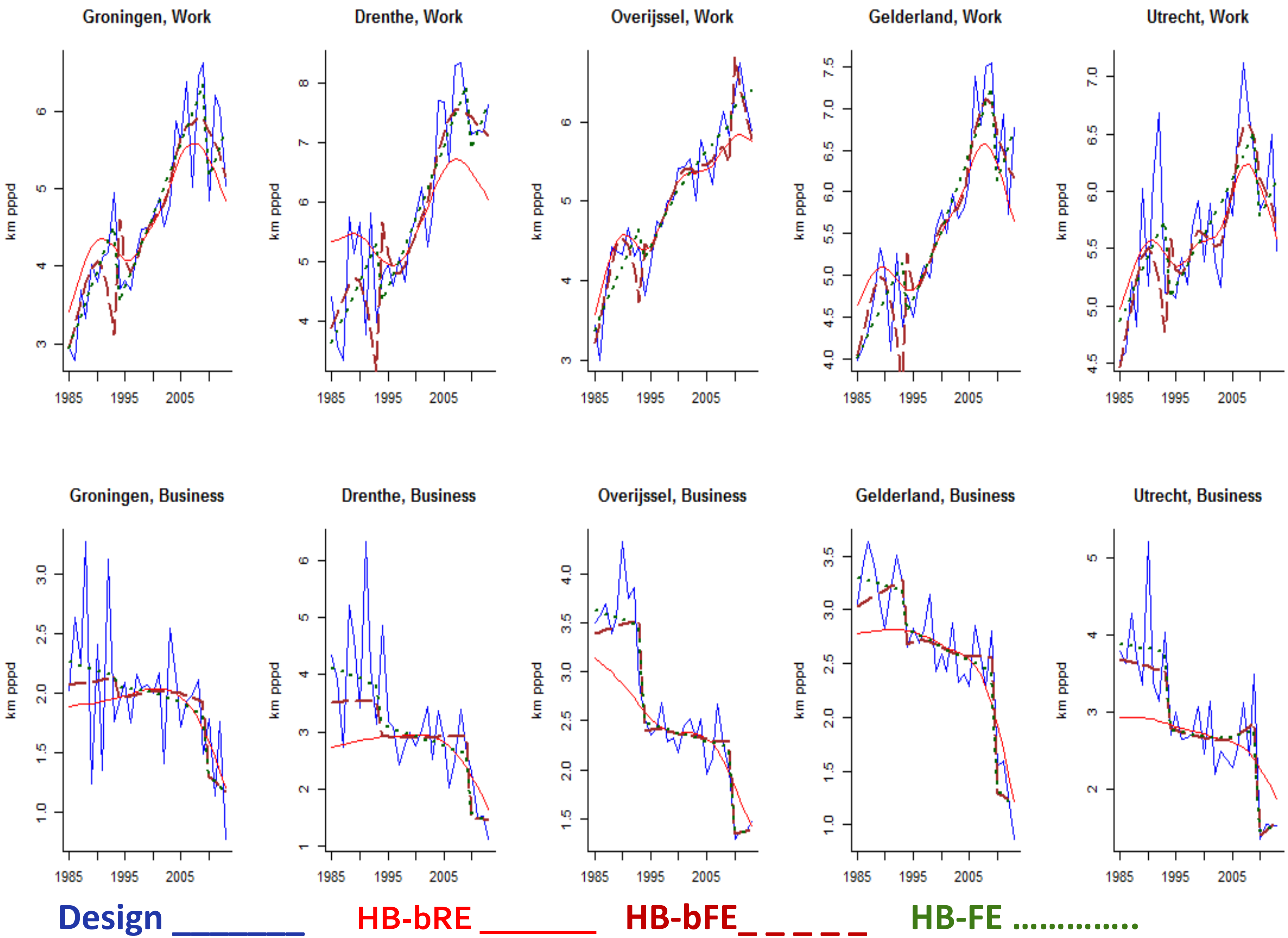
Smoothed survey error variance $\Phi_{m,t}$ can be replaced by $\sigma_{m,k}^{2}$ Var($Y_{m,t}$)



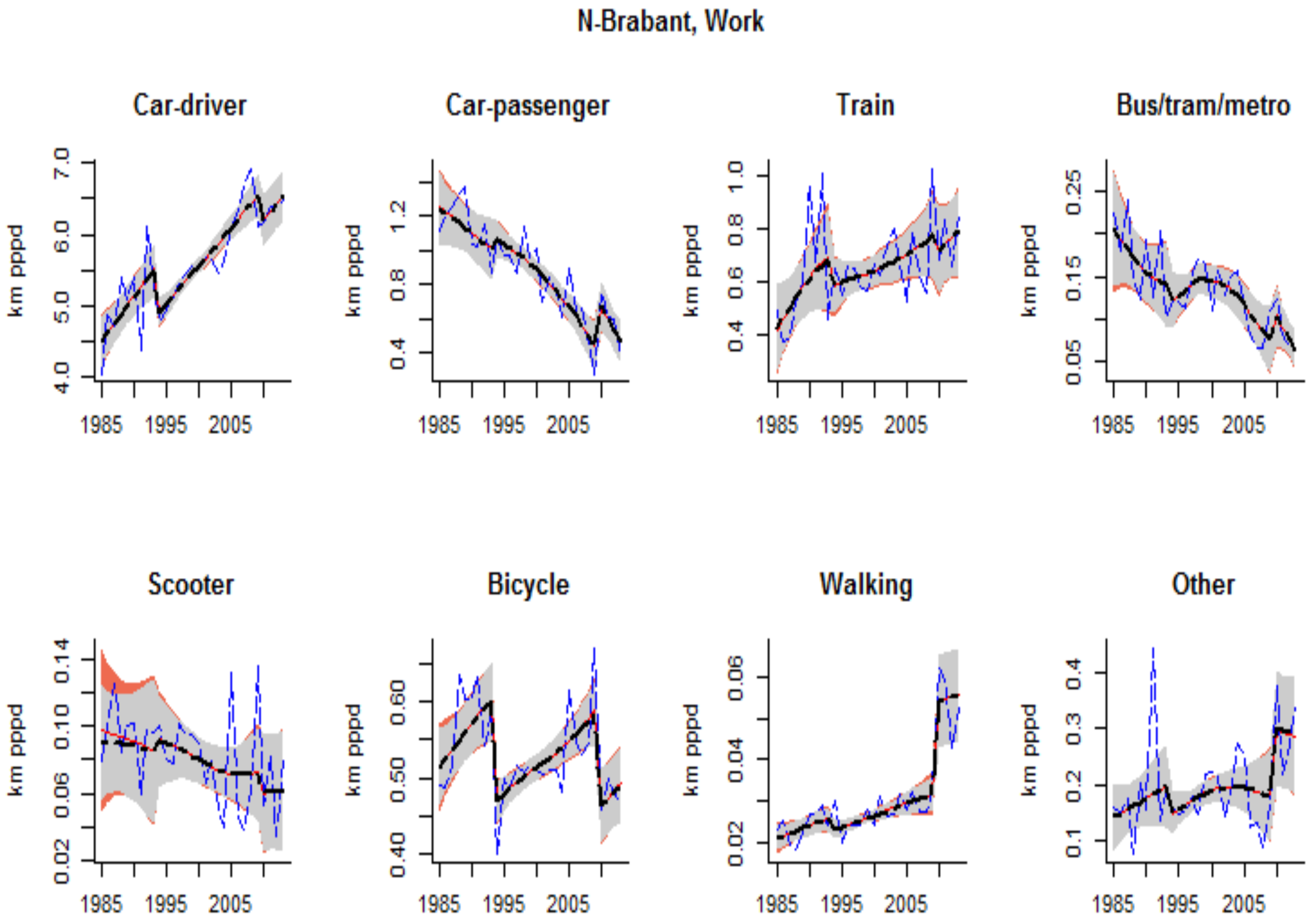
Provincial level Multilevel Models

Model Labels	Description	Iterations	DIC for 12 Provinces
HB-bRE	$\beta_{1994,m}$, $\beta_{2010,m}$ modelled as random effects	75000 (burn-in=50000, thinning=50)	-43668
HB-bFE	all $\beta_{k,m}$ modelled as fixed effects	55000 (burn-in=30000, thinning=50)	-43606
HB-FE	all $\beta_{k,m}$ AND intercepts ($c+v_m$) modelled as fixed effects	5000 (burn-in=1000, thinning=10)	-43859

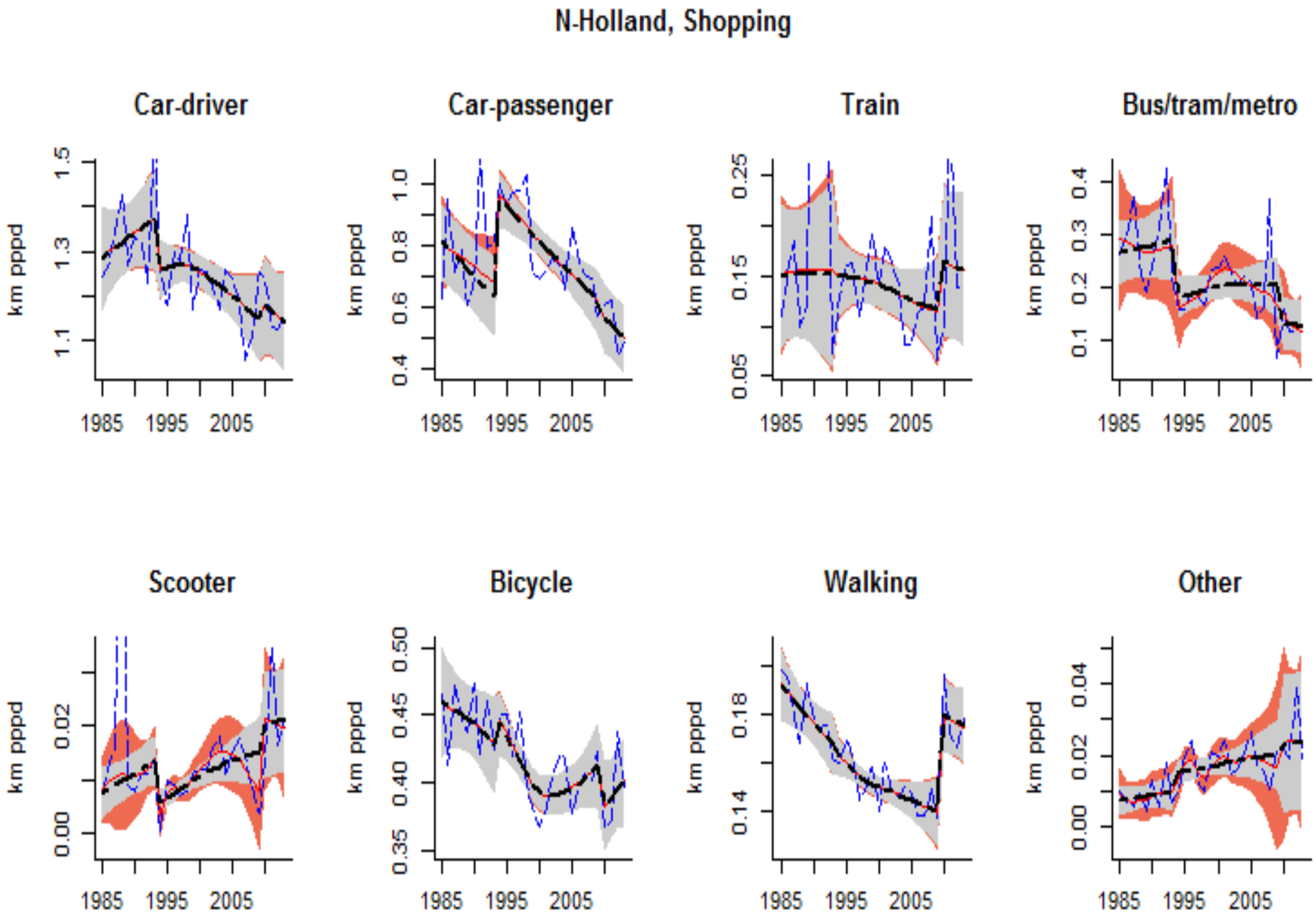
Car-driver



Groningen, Work
Drenthe, Work
Overijssel, Work
Gelderland, Work
Utrecht, Work
Groningen, Business
Drenthe, Business
Overijssel, Business
Gelderland, Business
Utrecht, Business
km pppd
1985
1995
2005
Design
HB-bRE
HB-bFE
HB-FE



Design -------- HB-FE --------- STS --------- (95% CIs in the same colour)



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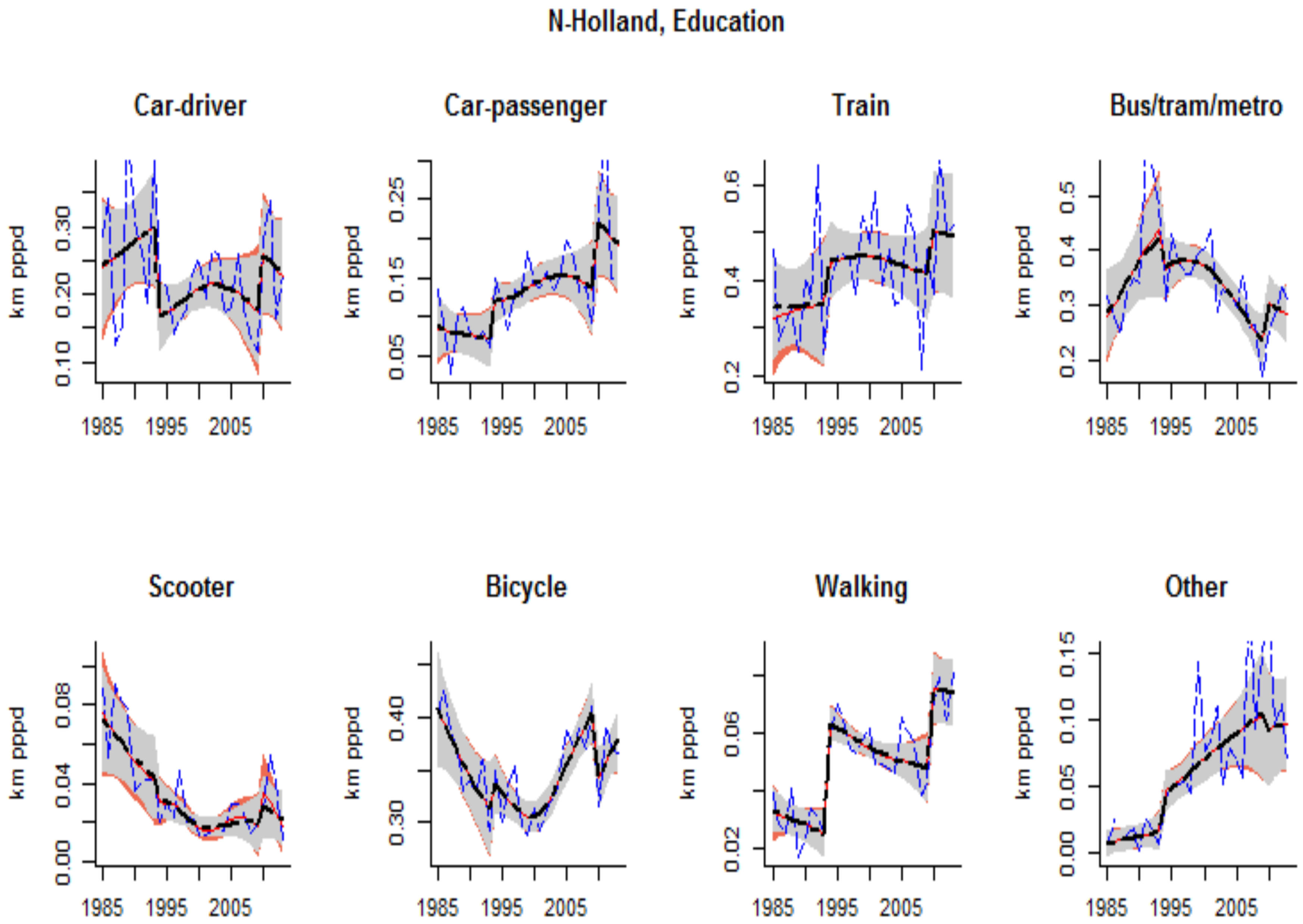
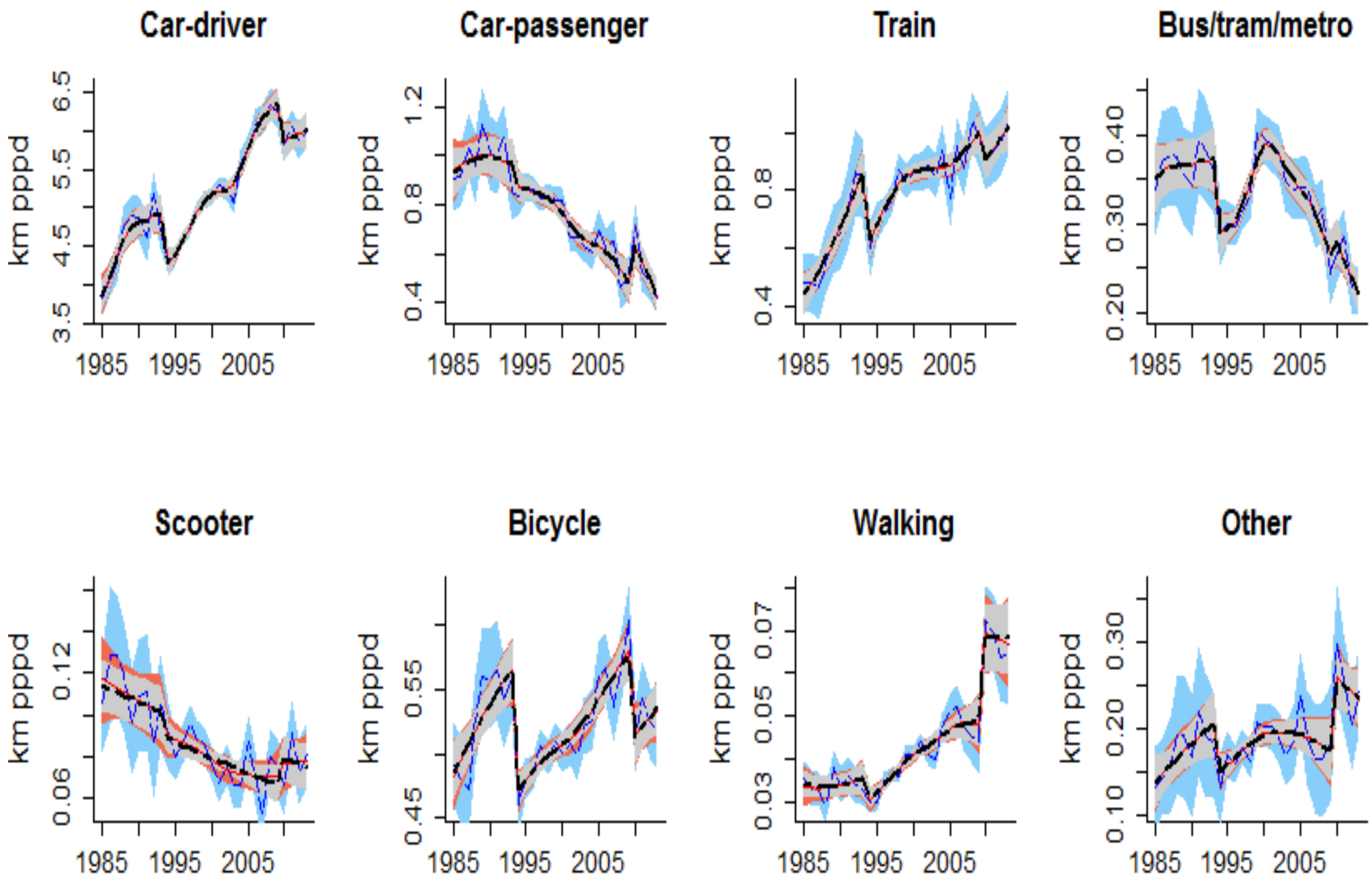


Design -------- **HB-FE ---------** **STS ---------** (95% CIs in the same colour)

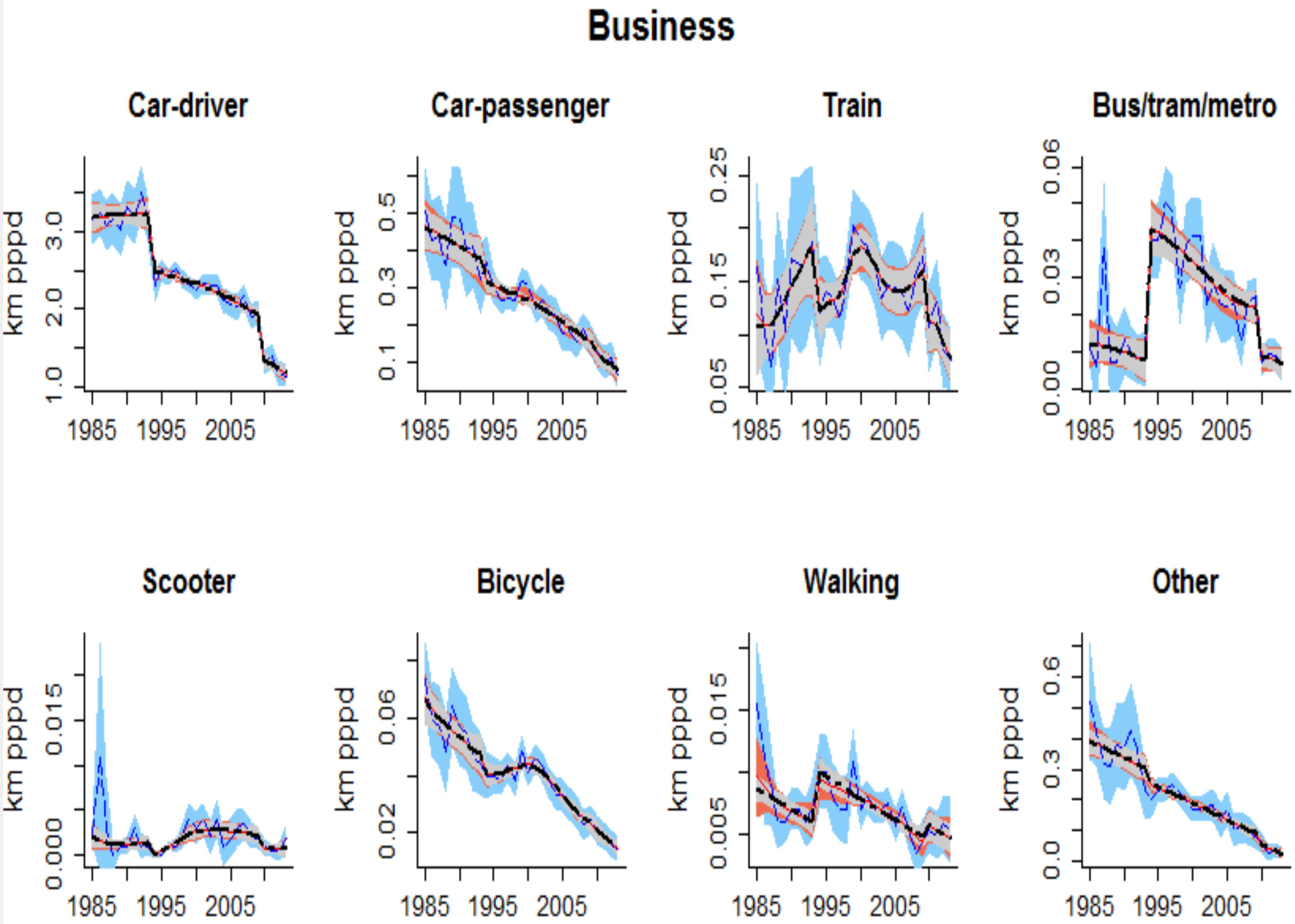
National level Multilevel Models

Model Labels	Description	Iterations	DIC for 12 Provinces
HB-bRE	$\beta_{1994,m}$, $\beta_{2010,m}$ modelled as random effects	75000 (burn-in=50000, thinning=50)	-8114
HB-bFE	all $\beta_{k,m}$ modelled as fixed effects	55000 (burn-in=30000, thinning=50)	-8166
HB-FE	all $\beta_{k,m}$ AND intercepts ($c+\nu_m$) modelled as fixed effects	5000 (burn-in=1000, thinning=10)	-8189

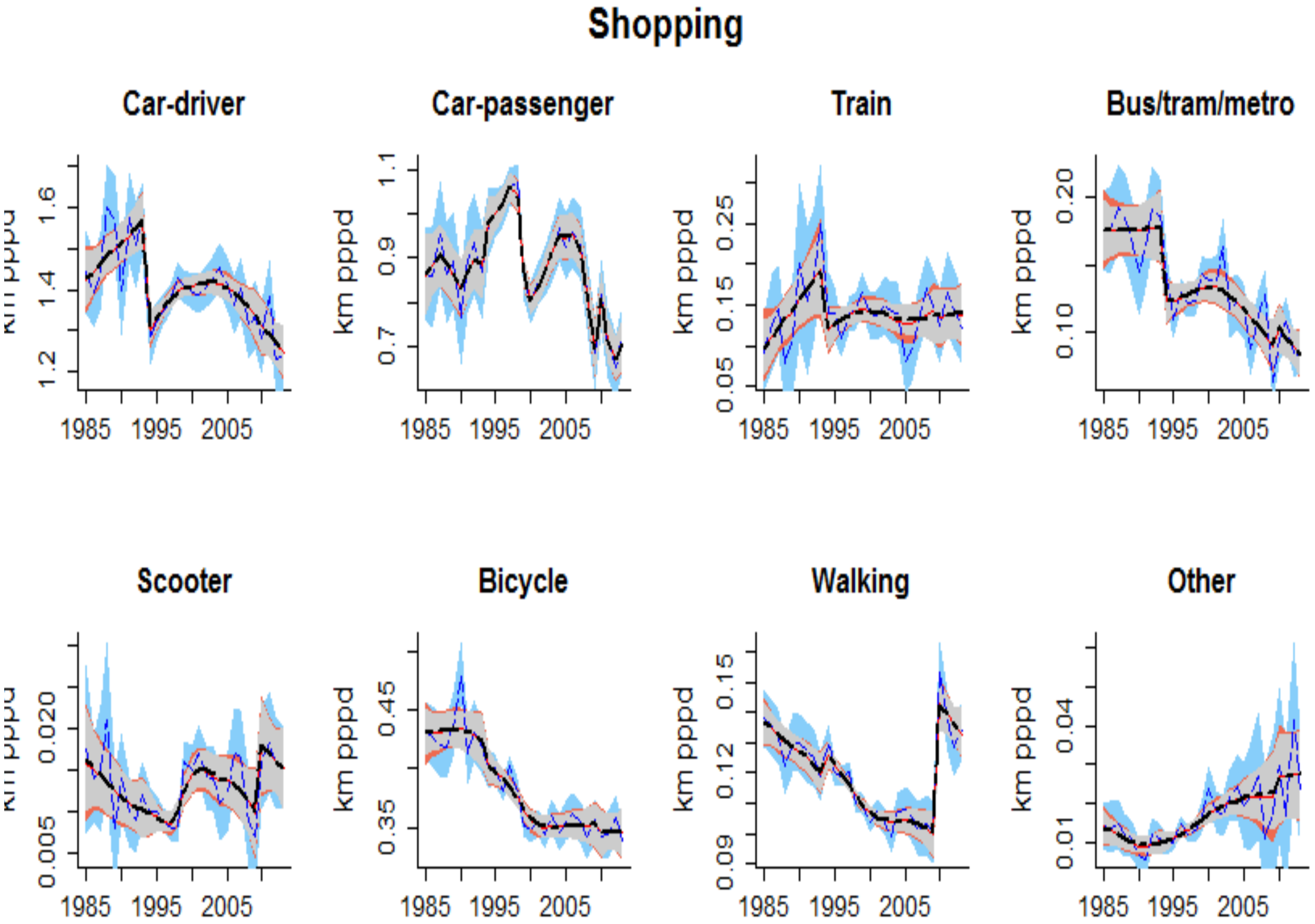
Work



Design -------- HB-FE --------- STS --------- (95% CIs in the same colour)

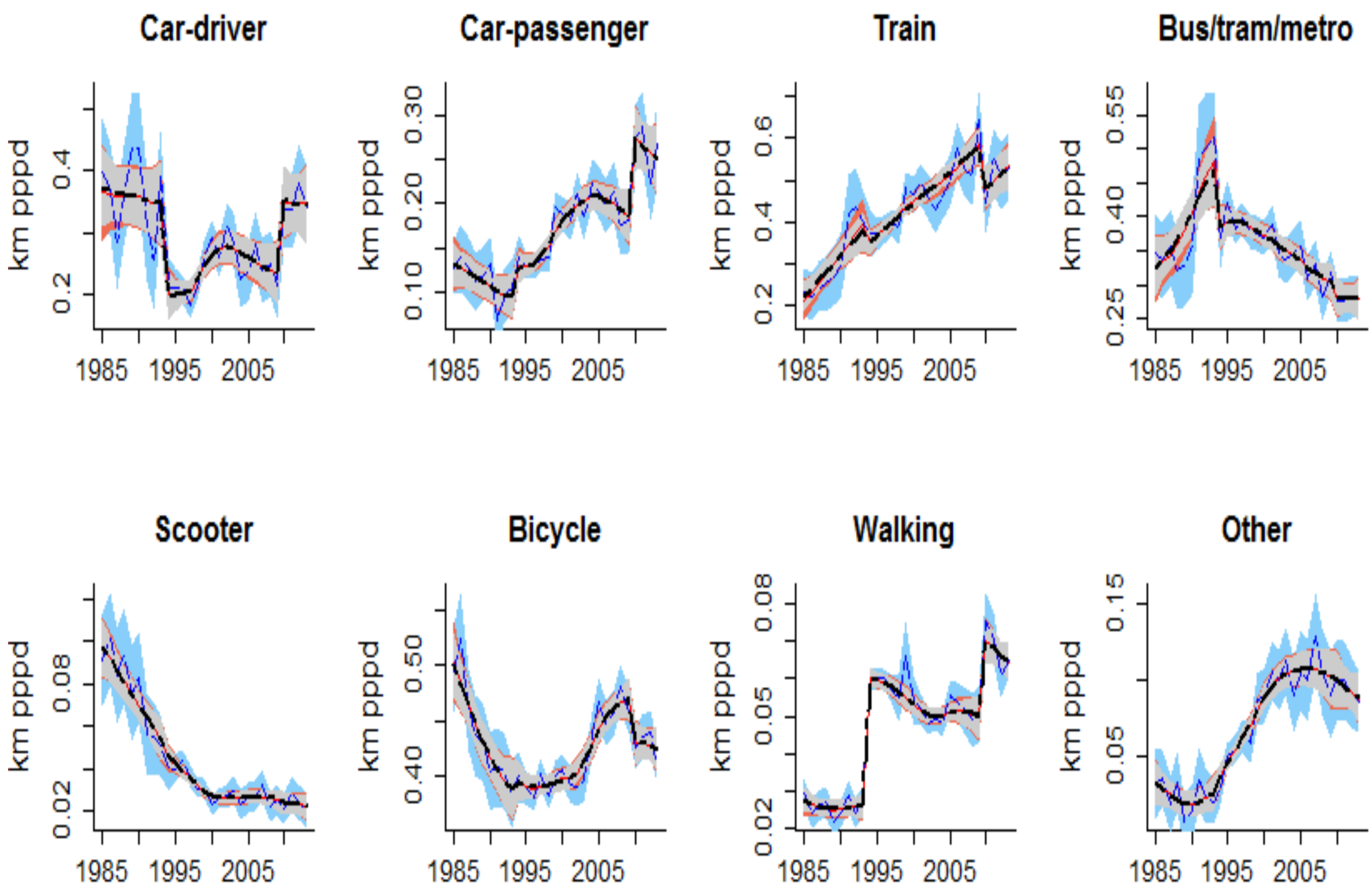


Design -------- HB-FE --------- STS --------- (95% CIs in the same colour)

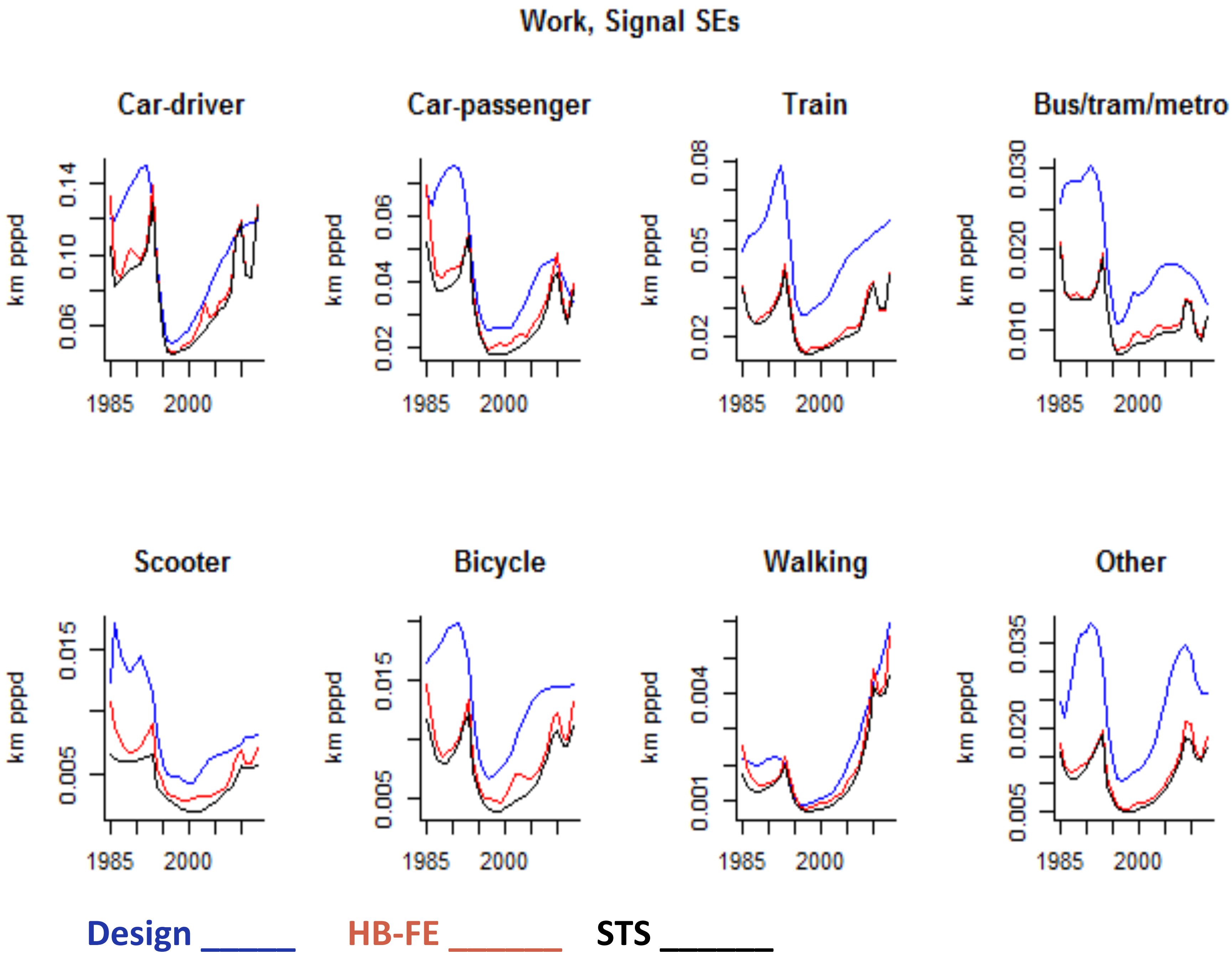


Design -------- HB-FE --------- STS --------- (95% CIs in the same colour)

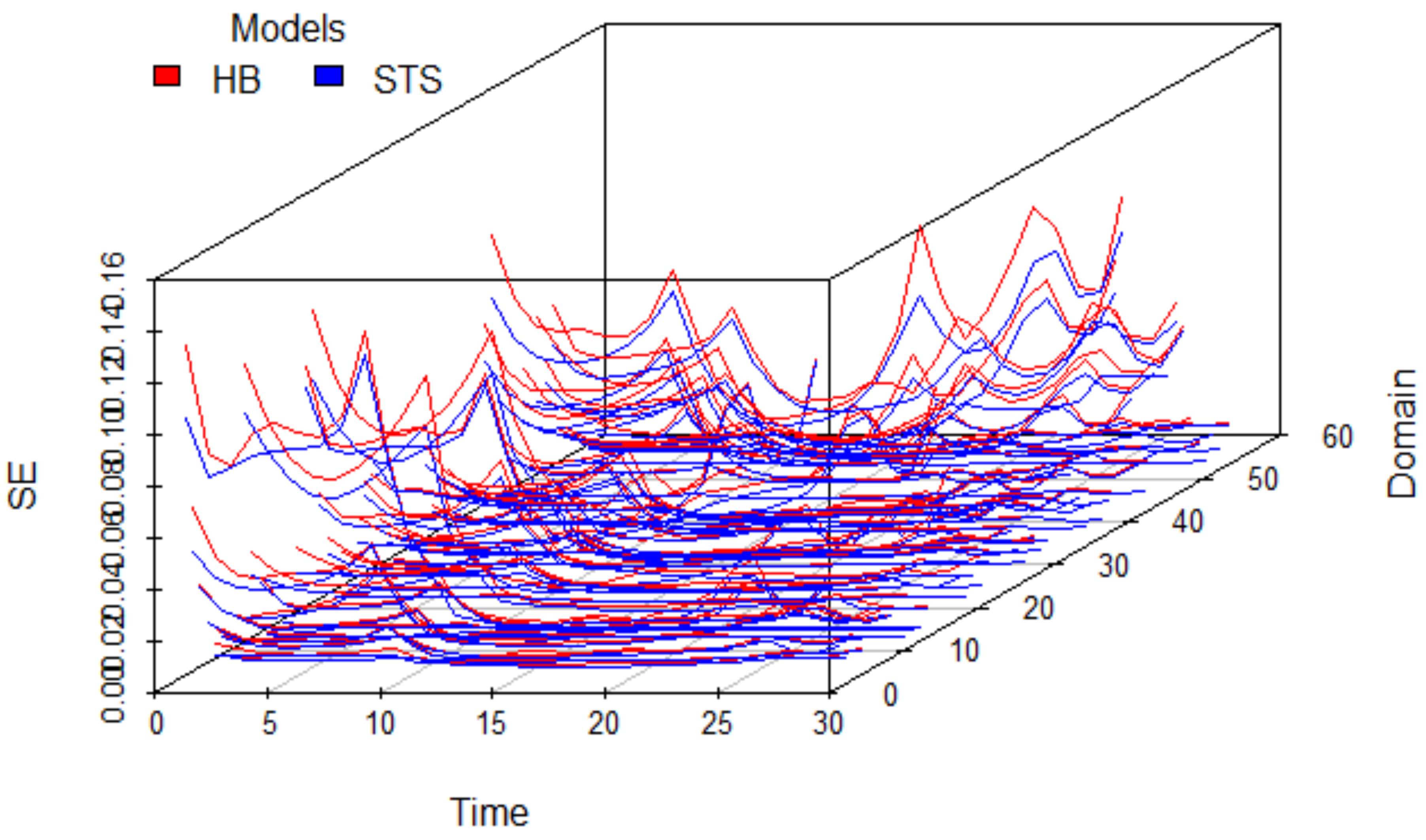
Education



Design -------- **HB-FE ---------** **STS ---------** (95% CIs in the same colour)


Work, Signal SEs
Car-driver
Car-passenger
Train
Bus/tram/metro
Scooter
Bicycle
Walking
Other
km pppd
1985
2000
0.06
0.10
0.14
0.02
0.04
0.06
0.02
0.05
0.08
0.010
0.020
0.030
0.005
0.015
0.001
0.004
0.005
0.020
0.035
Design
HB-FE
STS

Signal SEs, for 56 domains at the National level



Provincial level reduction in design SEs

– Percentage reduction in design SEs with **HB_FE** model:
  - **mean: 51%**
  - median: 54%

– Percentage reduction in design SEs with the multivariate **STS** model:
  - **mean: 54%**
  - median: 57%

$n'/n = 4.2$ in order to reach
$Var(Y)' = (1-0,51)\uparrow 2\ Var(Y)$

National level reduction in design SEs

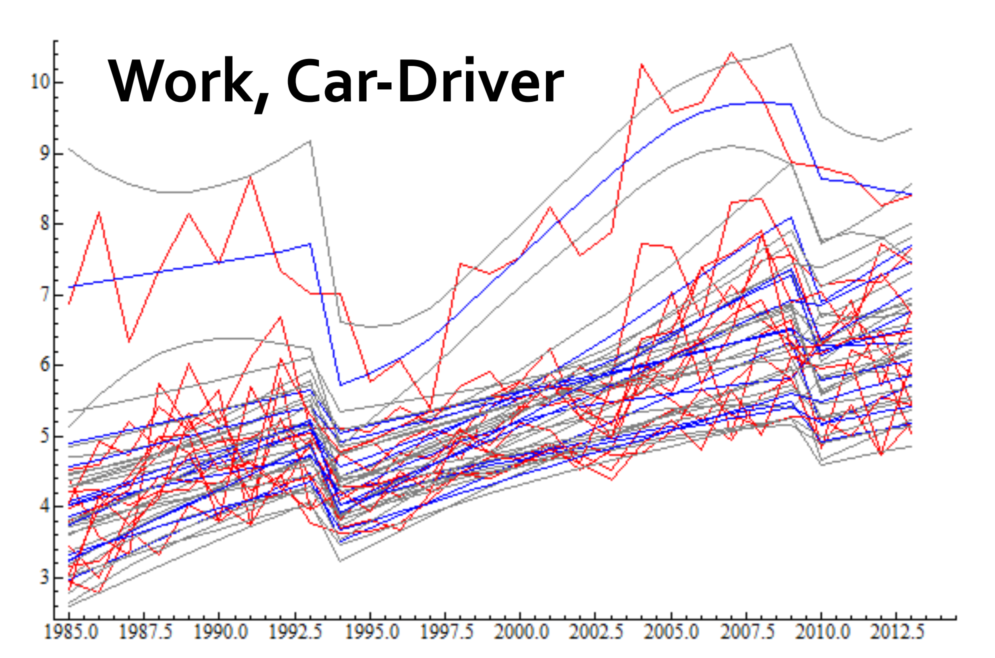
– Percentage reduction in design SEs with **HB_FE** model:
  - **mean: 32%**
  - median: 34%
– Percentage reduction in design SEs with the multivariate **STS** model:
  - **mean: 39%**
  - median: 41%

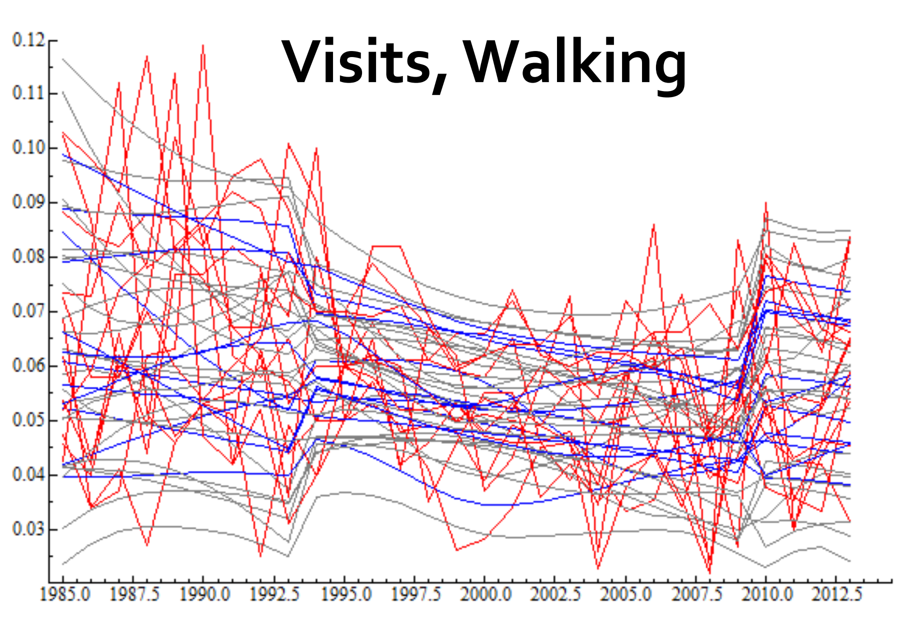
$n'/n = 2.2$

Conclusions

– PEs and SEs from HB and STS models are similar;
– signal SEs from HB models are somewhat higher;
– underestimation of the true signal variances in STS models does not seem to be significant in the DTS;
– multilevel approach relies on the STS one:
  - smoothing design SE estimates;
  - using survey error variances scaled within STS models.

Future research: Common Factor STS Models





OViN Redesign Planned for 2018

These time series models could be employed to:

- obtain decent variance estimates with shrinking sample sizes;
- estimate discontinuities due to the survey redesign.